Computer Science

Session of Wednesday
October 30, 2002

November 4, 2002

Review

Chomsky-hierarchy contains four classes of Languages [1] [2] [3] [4]

$$\mathcal{L}_{CH} \supsetneq \mathcal{L}_{CS} \supsetneq \mathcal{L}_{CF} \supsetneq \mathcal{L}_{reg.}$$

with according grammars:

$$\Gamma_{CH} \supsetneq \Gamma_{CS} \supsetneq \Gamma_{CF} \supsetneq \Gamma_{reg.}$$

$$\begin{array}{cccc} \Downarrow & \Downarrow & \Downarrow & \Downarrow \\ Turing & bounded & pushdown & finite \\ Machine & Turing & automaton & automaton \\ (TM) & Machine & pda & FA \end{array}$$

A Deterministic FA

is defined as 5-tupel

$$A = (Q, \Sigma, \delta, q_0, F)$$

with following

definitions Q is a finite(non-empty) set of states

Σ is an input alphabet

δ is a transition funtion of the form:

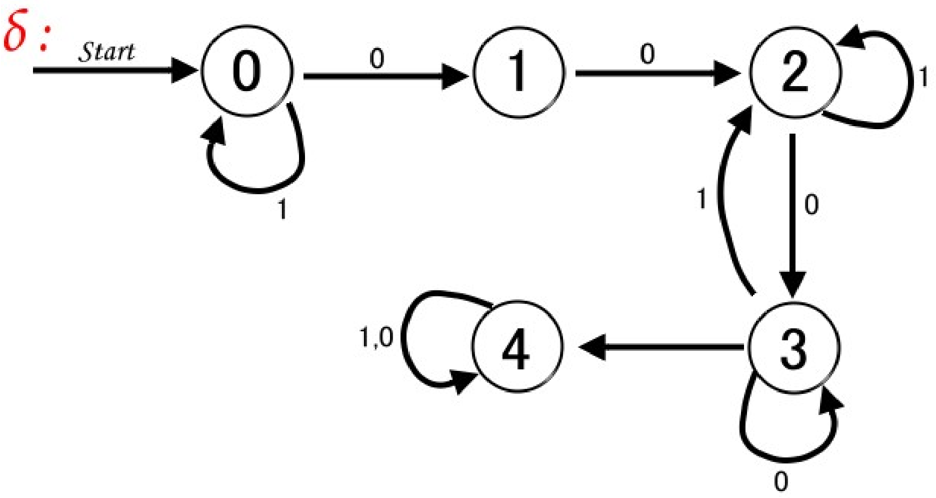
$$\delta : Q \times \Sigma \to Q$$

$q_0 \in Q$ is a starting state and

$F \subseteq Q$ is a set of finite sets.

[1] CH $\to$ CHomsky
[2] CS $\to$ Context Sensitive
[3] CF $\to$ Context Free
[4] reg. $\to$ regular

Example:
L={w ∈ {0,1}|* w has exact one pair of zeros → '00'}
$A = \{Q, \{0,1\}, \delta, F, q_0\}$



Q={0,1,2,3,4}   F={2,3} $q_0 \cong 0$

δ :

$\Sigma \setminus Q$	**0**	**1**	**2**	**3**	**4**
0	1	2	3	4	4
1	0	0	2	2	4

$\mathbf{A = \{\{0,1,2,3,4\},\{0,1\},\delta,0,\{2,3\}\}}$

Input string

1	1	0	0	1	0	

$\uparrow$ (input type)

Is $\delta(\mathbf{0}, \mathbf{110010})$ a word of the given language?

$$\begin{aligned}
\delta(0,110010) &= \delta(\delta(0,1),10010) \\
&= \delta(\delta(\delta(0,1),1),0010) \\
&= \delta(\delta(\delta(\delta(0,1),1),0),010) \\
&= \delta(\delta(\delta(\delta(\delta(0,1),1),0),0),10) \\
&= \delta(\delta(\delta(\delta(\delta(\delta(0,1),1),0),0),1),0) \\
&= \delta(\delta(\delta(\delta(\delta(\delta(\delta(0,1),1),0),0),1),0),\varepsilon)
\end{aligned}$$

A Non-deterministic FA

is defined as 5-tuple

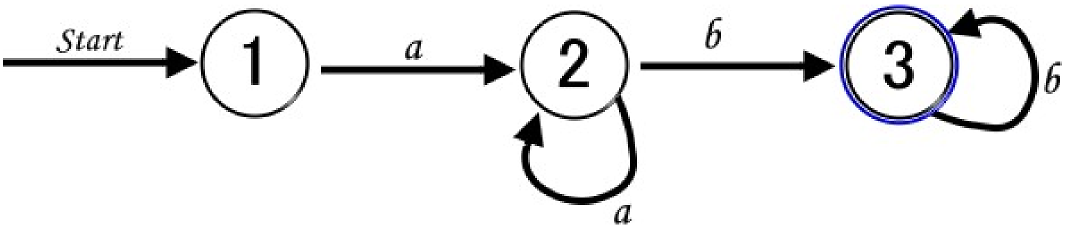
$$\mathbf{A} = (Q, \Sigma, \underline{\Delta}, q_0, F)$$

where a finite(non-empty) set of States $\mathbf{\Sigma}$ is an input alphabet.
$\mathbf{\Delta}$ is a transition relation of the form:

$$\mathbf{\Delta} : Q \times \Sigma^* \subseteq 2^{Q \times \Sigma^*}$$

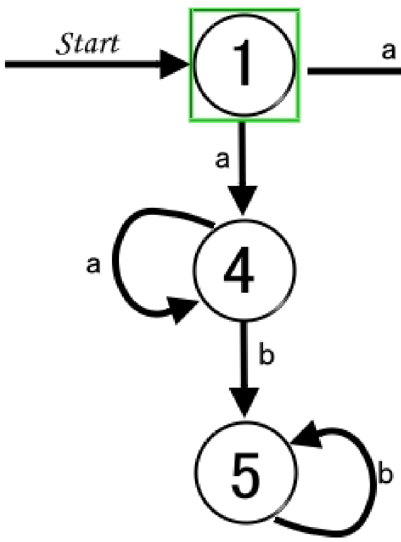
$\mathbf{q}_0 \in Q$ is a starting state
$\mathbf{F} \subseteq Q$ is a set of final states

deterministic FA



$\{w \in \{a,b\} | a^+b^+\} \quad \mathrm{L} = \mathrm{L(A)} \quad \mathbf{A} = (\{1,2,3\},\{a,b\},\delta,\{3\})$

non-deterministic FA



$\mathbf{L} = \{a,b\} \cup \{a^+b^+\} = \{a^+b^+\}$

The state '1' requires a uncertain decision.

	a	a	b	b	

↑

	a	b	

Representations for a $\mathbf{L} \in \mathcal{L}_{reg}$

- regular expressions

  $\Downarrow$ a regular language $L \in \mathcal{L}_reg$
- right or left linear grammar

- $\mathbf{L}(G), G \in \Gamma_{reg}$

  We can define a DFA[5] or nDFA[6] A'

  L(G)= L = L(A) $\Rightarrow$ L(G) = L(A)

  L(G)= L = L(A') $\Rightarrow$ L(G) = L(A')

Rightlinear grammar $\mathbf{G}, G \in \Gamma_{reg}$

that L(G) = L(A)
P:

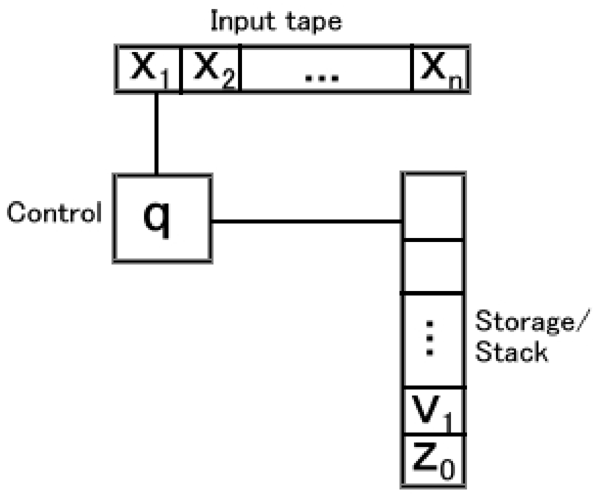
S $\rightarrow$ 1S | 0A
A $\rightarrow$ 1S | 0 | 0B
B $\rightarrow$ 1B | 0 | 0C
C $\rightarrow$ 1B | 1

G = (N,T,P,S) $\rightarrow$ G = ({S,A,B,C},{0,1},P,S)
reg expression

$(1^*001^*)\cup(000)\cup(1^*001^+0)\cup(1^*00(10)^*)$

[5]Deterministic Finite Automaton
[6]non-Deterministic Finite Automaton

Pushdown automaton(PDA)



non-Deterministic PDA

is defined as a 7-tuple

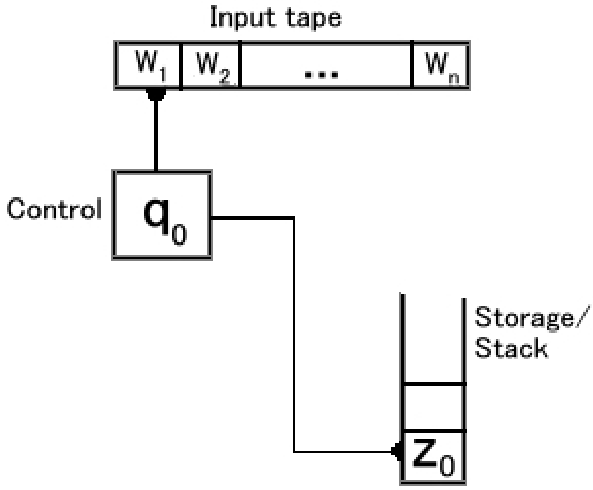
$$\aleph = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

where Q is a finite set of states
Σ is an input alphabet
Γ is a stack alphabet
δ is a transition relation of the form:

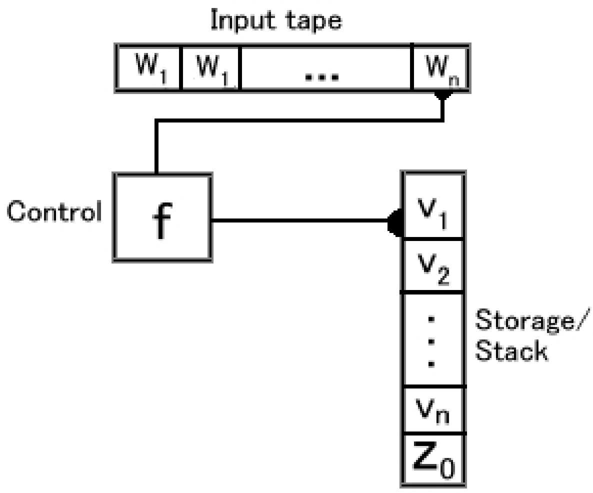
$$\delta \subseteq Q \times \Sigma \times \Gamma \times Q \times \Gamma^*$$

$q_0 \in Q$ is starting state
$Z_0 \in \Gamma$ is a stack symbol that marks the bottom of the stack
$F \subseteq Q$ is a set of final states

Start configuration



End configuration



$f \in F \subseteq Q$